1) A regular pentagon is centered about the origin and has a vertex at $(0,4)$. Which transformation maps the pentagon to itself?

D. a clockwise rotation of $144^{\circ}$ about the origin

$$
\frac{360^{\circ}<- \text { full circle }}{5<-\# \text { of identical sides }}=72^{\circ}
$$

Start multiplying $72^{\circ}$ to see which one is a multiple

$$
72^{\circ} * 2=144^{\circ}
$$

3) A parallelogram has vertices at $(0,0),(0,6),(4,4)$, and $(4,-2)$. Which transformation maps the parallelogram to itself?

C. a rotation of $180^{\circ}$ about the point $(2,2)$
4) What is the rule that describes the translation QRST $\rightarrow Q^{\prime} R^{\prime} S^{\prime} T^{\prime}$ ?

D. $(x, y) \rightarrow(x+6, y-2)$
5) If triangle $X Y Z$ is rotated $90^{\circ}$ clockwise about the origin to form triangle $X^{\prime} Y^{\prime} Z^{\prime}$, what are the coordinates of $Y^{\prime}$ ?

C. $(-2,-3)$
6) Look at quadrilateral QRST.


What is the image point $R$ after a clockwise rotation of 270 degrees about the origin?

Change 270 to 90 but the OTHER direction
C. $(-6,3)$
7) Parallelogram FGHJ was translated 3 units down to form parallelogram $F^{\prime} G^{\prime} H^{\prime} J^{\prime}$. Parallelogram $F^{\prime} G^{\prime} H^{\prime} J^{\prime}$ was then rotated $90^{\circ}$ counterclockwise about point $\mathrm{G}^{\prime}$ to obtain parallelogram $\mathrm{F}^{\prime \prime} \mathrm{G}^{\prime \prime} \mathrm{H}^{\prime \prime} \mathrm{J}^{\prime \prime}$.


Which statement is true about parallelogram FGHJ and parallelogram $\mathrm{F}^{\prime \prime} \mathrm{G}^{\prime \prime} \mathrm{H}^{\prime \prime} \mathrm{J}^{\prime \prime}$ ?

## A. The figures are both similar and congruent.

Congruent figures are AUTOMATICALLY similar figures
6) Which sequence of transformations maps $\triangle A B C$ to $\triangle R S T$ ?

B. Reflect $\triangle A B C$ across the line $\mathrm{x}=-1$. Then translate the result 5 units down.
$x=-1$ is a VUX (vertical line)
8)

## CONSTRUCTED RESPONSE

Rectangle WXYZ has coordinates $W(1,2), X(3,2), Y(3,-3)$, and Z(1, -3).
a. What would the coordinates be of $W^{\prime} X^{\prime} Y^{\prime} Z$ after a rotation of $90^{\circ}$ clockwise about the origin? Use the rule ( $y,-x$ )
$W^{\prime}(2,-1) X^{\prime}(2,-3) Y^{\prime}(-3,-3) Z^{\prime}(-3,-1)$
b. What would the coordinates be of $W^{\prime \prime} X^{\prime \prime} Y^{\prime \prime} Z^{\prime \prime}$ after a translation 2 units left and 3 units up? Use the rule $(x, y) \rightarrow(x-2, y+3)$
$W^{\prime \prime}(0,2) X^{\prime \prime}(0,0) Y^{\prime \prime}(-5,0) Z^{\prime \prime}(-5,2)$
c. Is rectangle $W X Y Z$ congruent to rectangle $W^{\prime \prime} X^{\prime \prime} Y^{\prime \prime} Z^{\prime \prime}$ ? Explain.

Yes, since rotations and translations create congruent figures since they are isometric transformations.

## Unit 2A: Properties of Angles, Triangles, and Quadrilaterals

9) What is the value of $x$ ?


Exterior $=$ interior + interior
$105=72+x$
A. 33
11) Lines $s$ and $t$ are parallel and $r$ is a transversal.


Which angles are congruent to $\angle 4$ ?
D. $\angle 3, \angle 7, \angle 8$
10) In the figure below, $\mathrm{p} \| \mathrm{q}$.


Which of these statements is NOT true?
C. $\boldsymbol{m} \angle \mathbf{6}+\boldsymbol{m} \angle \mathbf{3}=18 \mathbf{0}^{\circ}$ (these should be congruent since they have the same mark)
12) In the drawing below, line $h$ is parallel to line $k$.


What is the value of $y$ ?
$2 x+5=3 x-10$
$x=15$
$2(15)+5=35^{\circ}$
$180^{\circ}-35^{\circ}=145^{\circ}$
D. 145
13) In the drawing, line $p$ is parallel to line $j$ and line $t$ is perpendicular to $\overrightarrow{A B}$.


What is the measure of $\angle B A C$ ?

$$
180^{\circ}-53^{\circ}-90^{\circ}=37^{\circ} \quad \text { A. } 37^{\circ}
$$

15) Look at quadrilateral $A B C D$.


Which information is needed to show that quadrilateral ABCD is a parallelogram?
C. Use the slope formula to show that segments AB and CD have the same slope and segments AD and BC have the same slope.
17) Which of the following would be enough information to prove that quadrilateral QRST is a parallelogram?

14) Which information is needed to show that a parallelogram is a rectangle?
A. The diagonals bisect each other. (parallelogram)
B. The diagonals are congruent. (rectangle \& square)
C. The diagonals are congruent and perpendicular. (square)
D. The diagonals bisect each other and are perpendicular. (rhombus)
16) What proves that figure $A B C D$ is a parallelogram?
C. Diagonal BD bisects diagonal AC.

18) Quadrilateral RSTU has vertices $R(-6,-$ $3), S(3,3)$, and $T(4,-1)$. What are the coordinates of vertex $U$ if RSTU is a parallelogram?
B. (-5, -7)

Draw a triangle from $S$ to $T$ then go the same distance from $R$ to find $U$

19) A community is building a square park with sides that measure 120 meters. To separate the picnic area from the play area, the park is split by a diagonal line from opposite corners. Determine the approximate length of the diagonal line that splits the square. If necessary, round your answer to the nearest meter.

$(120)^{2}+(120)^{2}=(x)^{2} \quad 28800=x^{2} \quad 169.7 m=x$
B. 170 meters
21) $A B C D$ is a rhombus. How do you complete the explanation that states why $\triangle A B C \cong \triangle C D A$ ?

$\overline{A B} \cong \overline{C D}$ and $\overline{B C} \cong \overline{D A}$ by the definition of a rhombus. $\overline{A C} \cong$ $\overline{A C}$ by the Reflexive Property of Congruence, so $\triangle A B C \cong$ $\triangle C D A$ by $\qquad$ —.
D. SSS
20) Parallelogram $A B C D$ has vertices as shown.


Which equation would be used in proving that the diagonals of parallelogram ABCD bisect each other?
C. $\sqrt{(-1-1)^{2}+(4-0)^{2}}=\sqrt{(1-3)^{2}+(0+4)^{2}}$
22) Which set of statements would not be used to complete the proof?

A. $\triangle A B D \cong \triangle C B D$ by HL Congruence

## Unit 2B: Congruence

## Practice Problems

23) Consider the triangles shown.


Which can be used to prove the triangles are congruent?
D. AAS

The side is NOT between the angles
25) If $\triangle D N P \cong \triangle H K F$, which of the following is NOT true?
D. $\angle P \cong \angle K$
24) Rita is creating an abstract design that includes the figure below.


She knows that $\angle P Q R \cong \angle T S R$. What additional information would she need to prove that $\triangle P Q R \cong \triangle T S R$ using ASA?
Given two angles and need the side BETWEEN the angles
D. $\overline{Q R} \cong \overline{S R}$
26) In the proof below, what is the missing reason?

Given: $A B C D$ is a kite
Prove: $\angle B \cong \angle D$


| Statement | Reason |
| :--- | :--- |
| 1. $\overline{A B} \cong \overline{A D}$ and $\overline{B C} \cong \overline{C D}$ | 1. Definition of kite |
| 2. $\overline{A C} \cong \overline{A C}$ | 2. Reflexive Property of equality |
| 3. $\triangle A B C \cong \triangle A D C$ | 3. SSS |
| 4. $\angle B \cong \angle D$ | 4.? |

C. CPCTC (Parts that come AFTER proving triangles are congruent)
27) In the diagram, $\overline{C D}$ is the perpendicular bisector of $\overline{A B}$. The twocolumn proof shows that $\overline{A C}$ is congruent to $\overline{B C}$.


| Step | Statement | Justification |
| :---: | :--- | :--- |
| 1 | $\overline{C D}$ is the perpendicular bisector of $\overline{A B}$. | Given |
| 2 | $\overline{A D} \cong \overline{B D}$ | Definition of bisector |
| 3 | $\overline{C D} \cong \overline{C D}$ | Reflexive Property of Congruence |
| 4 | $\angle A D C$ and $\angle B D C$ are right angles. | Definition of perpendicular lines |
| 5 | $\angle A D C \cong \angle B D C$ | All right angles are congruent. |
| 6 | $\triangle A D C \cong \triangle B D C$ |  |
| 7 | $\overline{A C} \cong \overline{B C}$ | CPCTC |

Which of the following would justify Step 6?
C. SAS
30) In this diagram, $\overline{D E} \cong \overline{J I}$ and $\angle D \cong \angle J$.


Which additional information is sufficient to prove $\triangle D E F$ is congruent to $\Delta J I H$ ?

Match corresponding parts
C. $\overline{\boldsymbol{D H}} \cong \overline{J F}$
28) What other information is needed in order to prove the triangles congruent using the SAS Congruence Postulate?

B. $\overline{A C} \perp \overline{B D}$

Need an angle between
29) In this diagram, STU is an isosceles triangle where $\overline{S T}$ is congruent to $\overline{U T}$. The paragraph proof shows that $\angle S$ is congruent to $\angle U$.


It is given that $\overline{S T}$ is congruent to $\overline{U T}$. Draw $\overline{T V}$ such that $V$ is on $\overline{S U}$ and $\overline{T V}$ bisects $\angle T$. By the definition of an angle bisector, $\angle S T V$ is congruent to $\angle U T V$. By the Reflexive Property of Congruence, TV is congruent to $T V$. Triangle $S T V$ is congruent to triangle $U T V$ by SAS. $\angle S$ is congruent to $\angle U$ by $\qquad$ ? _.

Which step is missing in the proof?
PARTS come after triangles
A. CPCTC

## Unit 2C: Similarity

31) What transformation results in a figure that is similar to the original figure but has a greater area?
D. a dilation of $\triangle Q R S$ by a scale factor of 2

Only one with a scale factor greater than 1
33) Given: $\overline{A B} \| \overline{D E}$. Which can be used to prove $\triangle A B C \sim \triangle E D C$ ?


Mark alternate interior angles because of parallel lines

## A. AA Similarity Postulate

35) In the triangles shown, $\triangle A B C$ is dilated by a factor of $\frac{2}{3}$ to form $\triangle X Y Z$. (This means they are similar which means ALL corresponding angles are congruent)


Given that $m \angle A=50^{\circ}$ and $m \angle B=100^{\circ}$, what is $m \angle Z$ ? $180^{\circ}-50^{\circ}-100^{\circ}$
C. $30^{\circ}$
32) In the triangle shown, $\overline{G H} \| \overline{D F}$.


What is the length of $\overline{G E}$ ?
B. 4.5
34) Given: $\angle B \cong \angle D$. Which of the following conjectures can NOT be made based on the diagram and the given information?

C. $\overline{B D} \perp \overline{A C}$
36) A landowner wants to find the distance $d$ across a pond. The two overlapping right triangles shown are similar. Using the measurements shown, what is d ?

C. 96 meters
37) Figure $A^{\prime} B^{\prime} C^{\prime} D^{\prime} F^{\prime}$ is a dilation of figure $A B C D F$ by a scale factor of $\frac{1}{2}$. The dilation is centered at $(-4,-1)$.


Which statement is true?
B. $\frac{A B}{A^{\prime} B^{\prime}}=\frac{B C}{B^{\prime} C^{\prime}}$ Match the SAME parts (original points on top and primes on the bottom)
40) A bell tower is 17 meters tall. It casts a long shadow on the ground below. The tip of the shadow of the bell tower is 51 meters from the base of the bell tower. At the same time, a tall elm tree casts a shadow that is 63 meters long. If the right triangle formed by the tower and its shadow is similar to the right triangle formed by the elm and its shadow, how tall is the elm to the nearest tenth? Use corresponding parts (tall with tall, shadow with shadow and in the same order)

$$
\frac{17}{x}=\frac{51}{63}
$$

38) Which transformation on quadrilateral ABCD produces an image that does not preserve distance between points in quadrilateral ABCD?

## C. dilation by a scale factor of 2

Dilations change the size which means the distance between the points change
39) Helena creates similar (same scale factor) rectangles using exactly 100 cm of string. The smaller rectangle is 4 cm by 6 cm . What are the dimensions of the larger rectangle?

100 cm in total of string used $\rightarrow$ Small rectangle used $4+4+6+6=$ 20 cm of string so there is 80 cm left
A. 18 cm by $22 \mathrm{~cm}=80 \mathrm{~cm}$ perimeter, scale factor $\frac{18}{4}=4.5, \frac{22}{6}=3.7$
B. 20 cm by $30 \mathrm{~cm}=100 \mathrm{~cm}$ perimeter
C. 16 cm by $\mathbf{2 4} \mathbf{~ c m}=80 \mathrm{~cm}$ perimeter, scale factor $\frac{16}{4}=4, \frac{24}{6}=4$
D. 36 cm by $54 \mathrm{~cm}=90 \mathrm{~cm}$ perimeter
41) In triangle $A B C, \overline{P Q}$ is parallel to $\overline{B C}$. What is the length of $\overline{P B}$, in centimeters?

C. $\mathbf{2 0 ~ c m}$
42) This is a proof of the statement "If a line is parallel to one side of a triangle and intersects the other two sides at distinct points, then it separates these sides into segments of proportional lengths."


| Step | Statement | Reason |
| :---: | :--- | :--- |
| 1 | $\overline{G K}$ is parallel to $\overline{H J .}$ | Given |
| 2 | $\angle I G K \cong \angle I H J$ <br> $\angle I K G \cong \angle I H H$ | $?$ |
| 3 | $\triangle G I K \sim \triangle H I J$ | AA Similarity |
| 4 | $\frac{I G}{I H}=\frac{I K}{I J}$ | Corresponding sides of similar triangles are <br> proportional. |
| 5 | $\frac{H G+I H}{I H}=\frac{J K+I J}{I J}$ | Segment Addition Postulate |
| 6 | $\frac{H G}{I H}=\frac{J K}{I J}$ | Subtraction Property of Equality |

Which reason justifies Step 2?
C. Corresponding angles are congruent.

Angles are in the SAME location for both triangles
43) Look at the triangle.


Which triangle is similar to the given triangle?

44) In the coordinate plane, segment $\overline{P Q}$ is the result of a dilation of segment $\overline{X Y}$ by a scale factor of $\frac{1}{2}$. Draw lines through corresponding points


Which point is the center of dilation?
A. $(-4,0)$

## Constructed Response

46) 

Figure $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a dilation of figure $A B C D$.

A. Determine the center of dilation.
$(4,2)$
B. Determine the scale factor of the dilation.
Coming from the center of dilation $B^{\prime}$ went 1 right and $B$ went 2 right

$$
\frac{B^{\prime}}{B}=\frac{\mathbf{1}}{\mathbf{2}}
$$

C. What is the relationship between the sides of the pre-image and the corresponding sides of the image? The corresponding sides of the image are $1 / 2$ the size of the corresponding pre-image sides.

## Unit 3: Right Triangle Trigonometry

## WHAT YOU NEED TO KNOW

$\square \quad$ Pythagorean Theorem: $a^{2}+b^{2}=c^{2}$ ( $c$ is the hypotenuse)
$\square$ Trig Ratios: Sine $=\frac{o p p}{h y p} ; \underline{\text { cosine }}=\frac{a d j}{h y p} ;$ tangent $=\frac{o p p}{a d j}$
$\square$ Co-Functions: $\sin (\theta)=\cos (90-\theta) ; \cos (\theta)=\sin (90-\theta) ; \tan (\theta)=\frac{1}{\tan (90-\theta)}$
$\square$ Solving sides using trig ratios: Label all sides from the given angle (hypotenuse, opposite, adjacent) and circle the sides that have your important information and cross multiply
$\square$ Solving angles with trig inverses $\left({ }^{-1}\right.$ ): Label all sides from the given angle (hypotenuse, opposite, adjacent) and circle the sides that have your important information and use the inverse
$\square$ Angle of Elevation and Depression: Angle goes in the bottom of the triangle
Other Word Problems: Kites, ladders, etc.
$\square$ Square and Equilateral Triangles: Diagonals and altitudes

## Practice Problems

47) Triangle $A B C$ is given below.

48) In triangle $A B C$, angle $A$ and angle $B$ are complementary angles. The value of $\cos A$ is $\frac{5}{13}$. What is the value of $\sin B$ ?

$$
\cos A=\sin B \quad \text { A. } \frac{5}{13}
$$

49) Which equation is true? $\sin ($ angle $)=\cos (90-$ angle $)$
D. $\cos 40^{\circ}=\sin 50^{\circ}$
50) What is the approximate value of $x$ in the diagram below?
A. 7.6 cm

$$
\sin (25)=\frac{x}{18}
$$

52) In right triangle $H J K, \angle J$ is a right angle and $\tan \angle H=1$. Which statement about triangle HJK must be true?

Tangent is opposite over adjacent. If tangent = 1 , this means opposite and adjacent have to be the same. This means sine and cosine would be the same
C. $\sin \angle H=\cos \angle H$

51) At a certain time, a vertical pole 10 feet tall casts a 14-foot shadow. What is the angle of elevation of the sun to the nearest degree?

53) A hot air balloon is 1,200 feet above the ground. The angle of depression from the basket to the base of a monument is $54^{\circ}$.

Move your angle of depression into the elevation spot


$$
\sin (54)=\frac{1200}{d}
$$

Which equation can be used to find the distance, $d$, in feet, from the basket of the hot air balloon to the base of the monument?
B. $\sin 54^{\circ}=\frac{1200}{d}$
54) A 12 -foot ladder is leaning against a building at a $75^{\circ}$ angle with the ground.


Which equation can be used to find how high the ladder reaches up the side of the building?
D. $\sin 75^{\circ}=\frac{x}{12}$

Jane and Mark each build ramps to jump their remote-controlled cars. Both ramps are right triangles when viewed from the side. The incline of Jane's ramp makes a 30-degree angle with the ground, and the length of the inclined ramp is 14 inches. The incline of Mark's ramp makes a 45-degree angle with the ground, and the length of the inclined ramp is 10 inches.
Part A: What is the horizontal length of the base of each ramp? Explain how you found your answers. Write your answers below.

Jane: When I labeled all of the sides from the given angle on the ground, I had to use cosine since I wanted the bottom of the ramp (the adjacent side) and I knew the incline (hypotenuse). $\cos (30)=\frac{x}{14}$ so the height $=12.12$ inches.

Mark: When I labeled all of the sides from the given angle on the ground, I had to use cosine since I wanted the bottom of the ramp (the adjacent side) and I knew the incline (hypotenuse). $\cos (45)=\frac{x}{10}$ so the height $=5 \sqrt{2}$ or 7.07 inches
55) If $\Delta \mathrm{FGH} \sim \Delta \mathrm{KLM}$ which of the following must be TRUE?

Use corresponding angles and same ratio

A. $\tan \mathrm{G}=\boldsymbol{\operatorname { t a n }} \mathrm{L}$
57)

## Constructed Response

Study the triangle.


What is sin $x$ ? Explain how you determined your answer.
I needed the opposite side since sine is $\frac{\text { opposite }}{\text { hypotenuse }}$. I used Pythagorean Theorem to find the missing side, $(x)^{2}+(3)^{2}=$ $(5)^{2}$ and calculated the side to be 4 in . Therefore, $\sin x=\frac{4}{5}$

Part B: Which car is launched from the highest point? Explain your
reasoning. Write your answer below.
Jane: When I labeled all of the sides from the given angle on the ground, I had to use sine since I wanted the height (the opposite side) and I knew the incline (hypotenuse). $\sin (30)=\frac{x}{14}$ so the height $=7$ inches

Mark: When I labeled all of the sides from the given angle on the ground, I had to use sine since I wanted the height (the opposite side) and I knew the incline (hypotenuse). $\sin (45)=\frac{x}{10}$ so the height $=5 \sqrt{2}$ or 7.07 inches

Mark had a higher launch point.

## Unit 4A: Circle Angle Relationships and Theorems

## WHAT YOU NEED TO KNOW

$\square$ Circle Vocabulary
$\square$ Angle Relationships:

- Central (vertex is in the center) $\rightarrow$ equal to the arc measure
- 'ON' (vertex is on the edge circle) $\rightarrow$ equal to $1 / 2$ the arc measure
- 'IN'(vertex is in the circle, but NOT in the center) $\rightarrow \frac{\text { big arct small arc }}{2}=$ angle
- 'OUT' (vertex is outside of the circle) $\rightarrow \frac{\text { big arc-small arc }}{2}=$ angle


## $\square$ Chord Theorems

$\square$ Tangent Theorems: Radius and a tangent form a right angle; 2 tangents from the same point and go to the same circle are congruent

## Practice Problems

58) Circle $P$ is dilated to form circle $P^{\prime}$. Which statement is ALWAYS true?
If the radius of a circle is 2 , the circumference is $4 \pi$ If the radius of the dilated circle is 4 , the circumference is $8 \pi$

$$
\frac{4 \pi}{2}=2 \pi \text { and } \frac{8 \pi}{4}=2 \pi
$$

D. The ratio of the diameter to the circumference is the same for both circles.
59) In the circle shown, $\overline{B C}$ is a diameter and $m \overparen{A B}=120^{\circ}$. There is a diameter which makes $180^{\circ}$


What is the measure of $\angle A B C$ ? B. $30^{\circ}$
60) The figure shows Circle $C$ with tangent lines $\overleftrightarrow{S R}$ and $\overleftrightarrow{Q R}$. The measure of $\angle Q C S$ is $x^{0}$. Select THREE statements that are true about the figure.
A. The measure of $\angle Q P S$ is $\frac{1}{2} x^{\circ}$
B. The measure of $\angle C Q R$ is $90^{\circ}$ (radius and a tangent meet at a $90^{\circ}$ angle)
$E$. The measure of $\angle Q R S$ is $(180-x)^{\circ}$
Angle QRS is an out angle, the two arcs would be $x^{\circ}$ and $(360-x)^{\circ}$

$$
\begin{gathered}
\frac{(360-x)^{\circ}-x^{\circ}}{2}=\text { Angle } Q R S \\
\frac{360^{\circ}-2 x^{\circ}}{2}=(180-x)^{\circ}
\end{gathered}
$$


62) What is the value of x for $m \overparen{A B}=45^{\circ}$ and $m \overparen{C D}=42^{\circ}$ ?

B. 43.5
63) What is the measure of arc $A B$ ?

C. $110^{\circ}$
64) Points $A, B, C, D$, and $E$ are located on the circle $O$, as shown in this figure.

$180($ triangle $)=2 x+90+40$

$$
180=2 x+130
$$

$$
50=2 x
$$

$$
25=x
$$

The measure of $\overparen{C D}$ is $80^{\circ}$. What is the value of $x$ ?
65)

Constructed Response
$\angle P N Q$ is inscribed in circle O and $m \overparen{P Q}=70^{\circ}$.

A. What is the measure of $\angle P O Q$ ?
$70^{\circ}$ since $\angle P O Q$ is a central angle
B. What is the relationship between $\angle P O Q$ and $\angle P N Q$ ? $\angle P N Q=\frac{1}{2} \angle P O Q$
C. What is the measure of $\angle P N Q$ ?
$35^{\circ}$ since $\angle P N Q$ is an inscribed ("ON") angle

## Constructed Response

In circle P below, $A B$ is a diameter.


If $m \angle A P C=100^{\circ}$, find the following:
A. $m \angle B P C=80^{\circ}$ (central angle)
B. $m \angle B A C=40^{\circ}$ (inscribed "ON" angle)
C. $m \overparen{B C}=80^{\circ}$
D. $m \overparen{A C}=100^{\circ}$

## Unit 4B - Arc length/Sector Area

## WHAT YOU NEED TO KNOW

Arc Length $\rightarrow \frac{\text { arc length }}{\text { circumference }(2 \pi r)}=\frac{\text { angle }(\theta)}{360^{\circ}}$ OR arc length $=\frac{2 \pi r \theta}{360}$
Sector Area $\rightarrow \frac{\text { sector area }}{\text { Area }\left(\pi r^{2}\right)}=\frac{\text { angle }(\theta)}{360^{\circ}}$ OR sector area $=\frac{\pi r^{2} \theta}{360}$
67) Circle $E$ is shown.


$$
\begin{gathered}
\frac{x}{2 \pi(12)}=\frac{145}{360} \\
360 x=3480 \pi \\
x=\frac{29 \pi}{3}
\end{gathered}
$$

What is the length (use circumference) of $\overparen{C D}$ ?
C. $\frac{29}{3} \pi \mathrm{yd}$
69) A sprinkler rotates through $60^{\circ}$, watering a section of a field. The distance from the sprinkler to the farthest point reached is 3 meters. Approximately how many square meters (area) of the field is watered?


$$
\begin{gathered}
\frac{x}{\pi(3)^{2}}=\frac{60}{360} \\
360 x=540 \pi \\
x=4.7
\end{gathered}
$$

D. 4.7 square meters

## Practice Problems

68) Circle $Y$ is shown.


$$
\begin{gathered}
\frac{x}{\pi(9)^{2}}=\frac{285}{360} \\
360 x=23085 \pi \\
x=\frac{513 \pi}{8}
\end{gathered}
$$

What is the area of the shaded part $\left(360^{\circ}-75^{\circ}\right)$ of the circle?
D. $\frac{513}{8} \pi \mathrm{~cm}^{2}$
70) An athlete is running along a circular path (arc length) that has a diameter of 250 yards. The arc traveled by the athlete is $120^{\circ}$. Using 3.14 for $\pi$, how many yards did the athlete run? Round the answer to the nearest yard.

B. 262 yards

$$
\begin{gathered}
\frac{x}{2 \pi(125)}=\frac{120}{360} \\
360 x=30000 \pi \\
x=261.8
\end{gathered}
$$

71) The spokes of a bicycle wheel form 10 congruent central angles.

The diameter of the circle formed by the outer edge of the wheel is 18 inches.


$$
\begin{gathered}
\frac{360^{\circ}}{10}=36^{\circ} \quad \frac{x}{2 \pi(9)}=\frac{36}{360} \\
360 x=648 \pi \\
x=5.65
\end{gathered}
$$

What is the length, to the nearest 0.1 inch, of the outer edge (arc length) of the wheel between two consecutive spokes (one angle)?
B. 5.7 inches

## Unit 4C - Volume

## WHAT YOU NEED TO KNOW

Volume: Cavalieri's Principle, prisms $(B h \rightarrow(L W) h)$, pyramids $\left(\frac{1}{3} B h \rightarrow \frac{1}{3}(L W) h\right)$, cylinders $\left(\pi r^{2} h\right)$, cones $\left(\frac{1}{3} \pi r^{2} h\right)$, and spheres $\left(\frac{4}{3} \pi r^{3}\right)$
Composite Volume: Adding or multiplying multiple 3D figures
Geometric Modeling: Earth (sphere), human torso (cylinder), mountain (cone), bowl of cereal (hemisphere)
Population Density: $\frac{\text { population }}{\text { area }}$

## Practice Problems

72) This is a hand drawing of a mountain. Explain which geometric shape could be used to estimate the total amount of Earth the mountain is made of.

B. Cone (3D and round at the bottom)
73) What is the volume of a cylinder with a radius of 3 in and a height of $\frac{9}{2}$ in?

$$
V=\left(\pi(3)^{2}\right) * \frac{9}{2}
$$

A. $\frac{81}{2} \pi \mathrm{in}^{3}$
74) Jason constructed two cylinders using solid metal washers. The cylinders have the same height, but one of the cylinders is slanted as shown.


Which statement is true about Jason's cylinders?
D. The cylinders have the same volume because they have the same cross-sectional area at every plane parallel to the bases.
76) A square pyramid is packaged inside a box.


5 inches
The space inside the box around the pyramid is then filled with protective foam. About how many cubic inches of foam is needed to fill the space around the pyramid? Box - Pyramid
C. 83 cubic inches

$$
V=(5 * 5) 5-\frac{1}{3}(5 * 5) 5
$$

75) Find the difference (subtract) in the volume of the two spheres below.
B. $684 \pi$


$$
V=\frac{4}{3} \pi(9)^{3}-\frac{4}{3} \pi(6)^{3}
$$

77) A company needs to package this bell in a rectangular box.


What are the smallest dimensions (length, width, and height) the rectangular box can have so that the lid of the box can also close?
B. 6 inches long, 6 inches wide, and 8 inches high
78) A construction company is preparing 10 acres of land for a new housing community. The land contains large rocks that need to be removed. A machine removes 10 rocks from 360 square feet of land.

$$
1 \text { acres }=43,560 \text { square feet }
$$

How many rocks need to be removed from the 10 acres of land before the new housing community can be built?
C. 12,100 rocks

$$
\begin{aligned}
& 10 \text { acres }=435,000 \text { sq. feet } \\
& \text { (multiple the rate by } 10 \text { ) } \\
& \text { The wanted result goes on top } \\
& \text { (rocks) } \\
& \qquad \frac{10 \text { rocks }}{360 f t^{2}} * \frac{435,600 f t^{2}}{1}
\end{aligned}
$$

A sandcastle mold is in the shape of a cylinder with a diameter of 6 inches and a height of 8 inches. To the nearest cubic inch, how much sand will fit in the sandcastle mold? Explain how you determined your answer. In your explanation, use the word pi instead of the symbol $\pi$. Write your answer below.

We divided the diameter to find the radius. The radius is 3 inches. We used the cylinder volume formula and replaced $r=3$ and $h=8$. The amount of sand was equal to 226 cubic inches is $V=\left(\pi(3)^{2}\right) 8=226.2$.
79) Joe counts 250 peach trees on $25 \%$ of the land he owns. He determined that there are 10 trees for every 1,000 square feet of land. About how many acres of land does Joe own?

## 1 acre $=43,560$ square feet

## 1000 trees for $100 \%$ (multiple by 4) of the land

A. 2.3 acres

The wanted result goes on top ( $\mathrm{ft}^{2}$ )

$$
\frac{1000 \mathrm{ft}^{2}}{10 \text { trees }} * \frac{1000 \text { trees }}{1}=\frac{1000000}{10}=100000 \mathrm{ft}^{2}
$$

The wanted result goes on top (acres)

$$
\frac{1 \text { acre }}{43560 \mathrm{ft}^{2}} * \frac{100000 \mathrm{ft}^{2}}{1}=\frac{1000000}{43560}=2.3 \text { acres }
$$

## Constructed Response

A pyramid and rectangular prism have congruent bases and equal heights. Write a statement comparing the volume of the figures and explain your reasoning. Write your answer below.

Since a pyramid's volume formula is $V=\frac{1}{3} B h$, a prism's volume formula is $V=B h$, and the bases and heights of both figures are congruent, we expect the pyramid's volume to be $\frac{1}{3}$ the volume of the prism.

## WHAT YOU NEED TO KNOW

 $\left.-2 \rightarrow \frac{1}{2}\right)$, replace ' $m$ ', ' $x$ ', and ' $y$ ' to solve for ' $b$ ' to create a new linear equation
$\square$ Midpoint: $\left(\frac{x_{2}+x_{1}}{2}, \frac{y_{2}+y_{1}}{2}\right)$
$\square$ Partitioning: $\left(x_{1}+\frac{a}{a+b}\left(x_{2}-x_{1}\right), y_{1}+\frac{a}{a+b}\left(y_{2}-y_{1}\right)\right)$, MAKE SURE TO PAY ATTENTION TO WHAT POINT YOU ARE COMING FROM!
$\square$ Perimeter and Area: Use distance formula
$\square$ Conic Circles Properties: $(x-h)^{2}+(y-k)^{2}=r^{2}$, center. ( $\mathrm{h}, \mathrm{k}$ ), and radius $=\mathrm{r}$
$\square$ Creating Conic Circle Equations from different scenarios: Center \& radius, graph, general form (completing the square)

## Practice Problems

82) Given the points $\mathrm{P}(2,-1)$ and $\mathrm{Q}(-9,-6)$, what are the coordinates of the point on the directed line segment $\overline{P Q}$ that partitions $\overline{P Q}$ in the ratio $\frac{3}{2}$ ? $\left(2+\frac{3}{3+2}(-9-2),-1+\frac{3}{3+2}(-6-(-1))\right)$
B. $\left(-\frac{12}{5},-3\right)$
83) 

## Constructed Response

Given the points $A(-1,2)$ and $B(7,8)$, find the coordinate of the point $P$ on directed line segment $\overline{A B}$ that partitions $\overline{A B}$ in the ratio 1:3.

$$
\begin{gathered}
\left(-1+\frac{1}{1+3}(7-(-1)), 2+\frac{1}{1+3}(8-2)\right) \\
\left(\mathbf{1}, \frac{\mathbf{7}}{\mathbf{2}}\right)
\end{gathered}
$$

83) What is the equation of a line that is perpendicular to $y=\frac{1}{2} x-6$ and passes through the point $(6,4)$ ?
D. $y=-2 x+16$

OLD SLOPE $\rightarrow 1 / 2$
NEW SLOPE $\rightarrow-2$ (opposite reciprocal)

$$
\begin{aligned}
& 4=-2(6)+b \\
& 4=-12+b \\
& 16=b
\end{aligned}
$$

85) 

## Constructed Response

Find the area of rectangle $A B C D$ with vertices $A(-3,0), B(3,2)$, $C(4,-1)$, and $D(-2,-3)$.

First graph the rectangle. If you notice $A B$ and $C D$ are the lengths and AD and BC are the widths.

Find the length and width using distance formula and then $A=l w$

$$
\begin{gathered}
A B=\sqrt{(3-(-3))^{2}+(2-0)^{2}} \quad B C=\sqrt{(4-3)^{2}+(-1-2)^{2}} \\
A B=\sqrt{(6)^{2}+(2)^{2}} \quad B C=\sqrt{(1)^{2}+(-3)^{2}} \\
A B=\sqrt{40} \text { or } 2 \sqrt{10} \quad B C=\sqrt{40} \\
\boldsymbol{A}=\sqrt{\mathbf{4 0}}(\sqrt{\mathbf{1 0}})=\mathbf{2 0}
\end{gathered}
$$

86) An equation of a line $a$ is $y=-\frac{1}{2} x-2$.


Which is an equation of the line that is perpendicular to line $a$ and passes through the point $(-4,0)$ ?
D. $y=2 x+8$

$$
\text { OLD SLOPE } \rightarrow-1 / 2
$$

$$
\text { NEW SLOPE } \rightarrow 2 \text { (opposite reciprocal) }
$$

$$
\begin{aligned}
& 0=2(-4)+b \\
& 0=-8+b \\
& 8=b
\end{aligned}
$$

87) What is the equation of a circle that has a center $(-6,2)$ and radius of $\sqrt{106}$ ?
D. $(x+6)^{2}+(y-2)^{2}=106$
88) Which of the following is the correct equation for the circle shown below?


Center $(-1,2)$ and radius is 7
B. $(x+1)^{2}+(y-2)^{2}=49$
89) A manufacturer is designing a two-wheeled cart that can maneuver through tight spaces. On one test model, the wheel placement (center) and radius is modeled by the equation $(x+2)^{2}+(y-1)^{2}=4$. What is the graph that shows the position and radius of the wheels? Center $(-2,1)$ and radius is 2
a.

b.

c.

d.

90) Which point is NOT on a circle with a center of $(0,0)$ and a radius of 10 ?

Equation: $x^{2}+y^{2}=100$
Plug in $x$ and $y$ until you do NOT get 100
A. $(0,5)$
$(0)^{2}+(5)^{2}=25($ not 100$)$
92) What is the center of the circle given by the equation $x^{2}+y^{2}-10 x-11=0$ ?
$x^{2}-10 x+$ $\qquad$ $+y^{2}=11+$ $\qquad$ $-\frac{10}{2}=(-5)^{2}=25$ $x^{2}-10 x+25+y^{2}=11+25$
$(x-5)^{2}+y^{2}=36$
A. $(5,0)$
94) Study this equation of a circle: $x^{2}-6 x+y^{2}+2 y+6=0$

$$
\begin{aligned}
& x^{2}-6 x+\ldots+y^{2}+2 y+\ldots=-6+ \\
& -\frac{6}{2}=(-3)^{2}=9 \quad \frac{2}{2}=(1)^{2}=1 \\
& x^{2}-6 x+\mathbf{9}+y^{2}+2 y+\mathbf{1}=-6+\mathbf{9}+\mathbf{1} \\
& (x-3)^{2}+(y+1)^{2}=4
\end{aligned}
$$

$\qquad$ $+$ $\qquad$

Which of these represents the center and radius of the circle?
C. center: (3, -1), radius: 2
91) Which point is on a circle with a center of $(3,-9)$ and a radius of 5 ?

Equation: $(x-3)^{2}+(y+9)^{2}=25$
Plug in $x$ and $y$ until you do 25
D. $(6,-5) \quad(6-3)^{2}+(-5+9)^{2}=25$
93) The circle shown below is centered at the origin $(0,0)$ and contains the point ( $-4,-2$ ).


Radius is from center to edge of the circle. Use the distance formula to find the radius.
$\sqrt{(0-(-4))^{2}+(0-(-2))^{2}}$
$2 \sqrt{5} * 2=$ diameter

Which of the following is closest to the length of the diameter of the circle?
C. 8.94
95) Which is an equation for the circle with a center at $(-2,3)$ and a radius of 3 ?

- Create the circle equation:

$$
(x+2)^{2}+(y-3)^{2}=(3)^{2}
$$

- Write groups out twice according to the exponent:

$$
(x+2)(x+2)+(y-3)(y-3)=9
$$

- Multiply out the groups:

$$
x^{2}+4 x+4+y^{2}-6 y+9=9
$$

- Group similar exponents together and combine like terms to one side
C. $x^{2}+y^{2}+4 x-6 y+4=0$


## Unit 6 - Applications of Probability

## WHAT YOU NEED TO KNOW

Sample Space: All possible outcomes (rolling a die - \{1, 2, 3, 4, 5, 6\}; flipping a coin - \{Heads, Tails $\}$ )
Total Possible Outcomes: \# of possible outcomes multiplied from ALL possible scenarios
Simple probability: $\frac{\text { wanted outcomes }}{\text { all outcomes }}$
Set Notation: Union (U) - OR, Intersection ( n ) - AND, Complement (') - NOT
'AND' events: $P(A$ and $B)=P(A) * P(B)$, independent events (with replacement - denominator doesn't change), dependent events (without replacement - denominator changes)
$\square \quad$ 'OR' events: $\mathrm{P}(\mathrm{A}$ or B$)=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}) * \mathrm{P}(\mathrm{B})$ (if needed), mutually exc/usive - no overlapping outcomes (no subtraction needed), inclusive overlapping outcomes (subtraction needed since the events have something in common), REMEMBER: you must subtract also if there are multiple sample spaces (flipping a coin and rolling a die)
$\square$ Conditional events: Based on a condition, $P(A \mid B)=\frac{P(A a n d B)}{P(B)}$, Event B is ALREADY happened
$\square$ Two-Way Frequency Tables: Pay attention to wording and the correct totals, relative frequency $\rightarrow$ percentage

## Practice Problems

96) For which set of probabilities would events $A$ and $B$ be independent?

Multiply $P(A)$ and $P(B)$ until it equals $P(A$ and $B)$
D. $P(A)=0.3 ; P(B)=0.15 ; P(A$ and $B)=0.045$
98) Each letter of the alphabet is written on separate cards in red ink. The cards are placed in a container. Each letter of the alphabet is also written on separate cards in black ink. The cards are placed in the same container. What is the probability that a card randomly selected from the container has a letter written in black ink or the letter is $A$ or $Z$ ?
$P($ black OR A OR Z $)=P($ black $)+P(A)+P(Z)-P($ black $A)-P($ black Z $)$ $\frac{26}{52}+\frac{2}{52}+\frac{2}{52}-\frac{1}{52}-\frac{1}{52}$
B. $\frac{7}{13}$
97) Bianca spins two spinners that have four equal sections numbered 1 through 4. If she spins a 4 on at least one spin, what is the probability that the sum of her two spins is an odd number?
$\left.\begin{array}{l|l|l|l|l|} \\ P(o d d ~ s u m ~ A N D ~ a t ~ l e a s t ~ o n e ~ s p i n ~ i s ~ a ~\end{array}\right) \quad 4,1$ =
$P($ one spin is at least a 4$)=$

| 4,1 | 4,2 | 4,3 | 4,4 |
| :--- | :--- | :--- | :--- |
| 1,4 | 2,4 | 3,4 |  |

$P$ (odd sum | at least one spin is a $4 \rightarrow$ odd sum GIVEN at least one spin is a 4$)=\frac{4}{7}$
C. $\frac{4}{7}$
99) Mrs. Klein surveyed 240 men and 285 women about their vehicles. Of those surveyed, 155 men and 70 women said they own a red vehicle. If a person is chosen at random from those surveyed, what is the probability of choosing a woman or a person who does NOT own a red vehicle?

$$
P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B)
$$

$P($ female $)+P($ not owning a red car) $-P($ females that don't own red cards)

$$
\frac{285}{525}+\frac{300}{525}-\frac{215}{525}
$$

C. $\frac{74}{105}$
100) In a particular state, the first character on a license plate is always a letter. The last character is always a digit from 0 to 9.

If $V$ represents the set of all license plates beginning with a vowel, and $O$ represents the set of all license plates that end with an odd number, which license plate belongs to the set V and O '?

V and NOT O would be start with a vowel and end in an even
A.

## E23 PCB C. ARB BKA

B.

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D.

## 52445

102) A random survey was conducted about gender and hair color. This table records the data.

Hair Color

|  | Brown | Blonde | Red | Total |
| :--- | ---: | ---: | ---: | ---: |
| Male | 548 | 876 | 82 | 1,506 |
| Female | 612 | 716 | 66 | 1,394 |
| Total | 1,160 | 1,592 | 148 | 2,900 |

What is the probability that a randomly selected person has blonde hair, given that the person selected is a male?

$$
\frac{876 \text { (blonde AND male) }}{1506 \text { (total male) }}
$$

C. 0.58
101) When rolling a fair, six-sided number cube, what is the probability of rolling an even number or a number less than 3 ?

3 even numbers $\{2,4,6\}+2$ numbers less than $3\{1,2,3\}-1$ number that is an even AND less than $3\{2\}$

$$
\frac{3}{6}+\frac{2}{6}-\frac{1}{6}
$$

B. $\frac{2}{3}$
103) Assume that the following events are independent:

- The probability that a high school senior will go to college is 0.72 .
- The probability that a high school senior will go to college and live on campus is 0.46 .

What is the probability that a high school senior will live on campus, given that the person will go to college?

Conditional probability

$$
\frac{P(H S \text { senior AND lives on campus })}{P(\text { goes to college })}=\frac{0.46}{0.72}
$$

D. 0.64
104) What is the probability of rolling a 5 on a fair, six-sided number cube if you know (given) that you rolled an odd number?

$$
\frac{1(\text { only } 15 \text { on a die })}{3(3 \text { odd numbers on a die) }}
$$

B. $\frac{1}{3}$
106)

## Constructed Response

A student draws a card from a standard deck and then draws another card without replacing the first card. Explain why the probability of picking an ace on the first draw and the probability of picking a 7 on the second draw are NOT independent events.

You are not replacing the Ace, so this would change the denominator for the $\mathbf{2}^{\text {nd }}$ card from 52 to 51 . This would increase the chance of drawing a $\mathbf{7}$ for the $2^{\text {nd }}$ event.
105) There are 5 pink marbles, 6 blue marbles, and 8 green marbles. You randomly pick two marbles out of the bag without looking. You chose a pink marble first and a green marble second. What was the probability of choosing the green marble as the second marble? (only want the probability of the $2^{\text {nd }}$ event)

$$
\frac{8(\text { still } 8 \text { green marbles left) }}{18 \text { (only } 18 \text { left after choosing pink) }}
$$

C. $\frac{4}{9}$

